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Relations between Domination and 2-Domination of Some Special Graph of Circulant Graph Families

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ABSTRACT

Let G = (V, E) be a simple, Connected, Undirected, finite graph without loops and multiple edges. In this paper we investigate bounds for the domination number and 2- domination number and its relationship with other parameters like strong (weak) domination number and independent strong (weak) domination number of some special graph of Circulant graph families.

Keywords: Domination, 2-Domination, Independent domination, Independent strong domination, Inverse domination, Circulant graphs.

1. INTRODUCTION

As Hedetniemi and Laskar (1990) note, the domination problem was studied from 1950’s onwards, but the rate of research on domination significantly increased in the mid-1970. Domination number of the graph is given by the minimum cardinality of dominating set of the graph. 2-Domination number of the graph is the minimum cardinality of 2-dominating set of graph. Here we discussed some parameters of domination and 2-domination of the graph. In Graph theory the dominating set for a graph G=(V, E) is a subset D of V such that every vertex not in D is adjacent to at least one member of D. Some parameters of domination are Inverse domination, Accurate domination, Strong domination, Independent domination, Independent strong domination. We discussed about the Domination number, Inverse domination number, Strong domination number, Independent domination number, Independent strong domination number.

2. DOMINATION OF A GRAPH

Some basic definitions and Notations:

Definition 2.1 [1]

A Dominating set of a graph G=(V,E) is a subset D of V such that every vertex not in D is adjacent to at least one vertex of D. The domination number **γ(G)** is the number of vertices in smallest dominating set of G.

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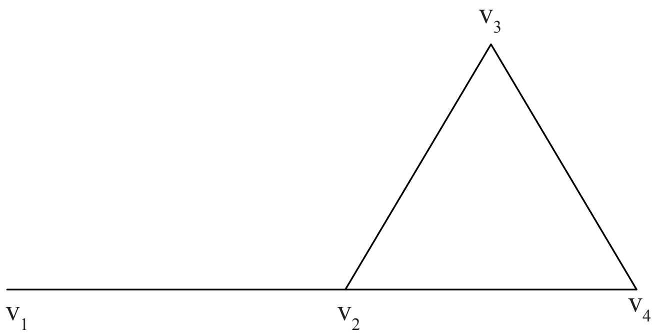
Example 2.1:

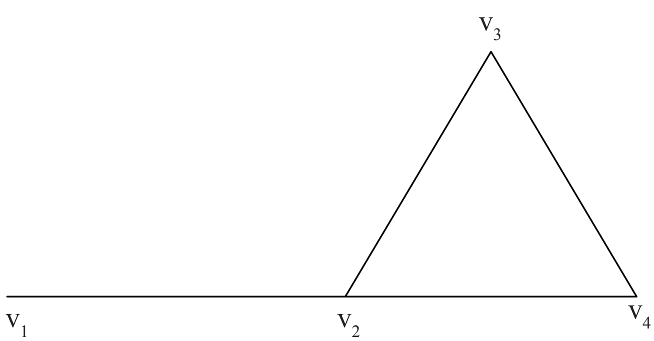


D={v_2} is the minimum dominating set &|D|=1 and γ(G)=1

Definition: 2.2 [2]

Let D be a dominating set of graph G. A dominating set D′⊆ V-D is called an Inverse Dominating set of G with respect to D. The Inverse Domination number γ′(G) of G is the cardinality of a smallest inverse Dominating set of G. The Minimum cardinality of D′(G) is the inverse domination number and is denoted by γ′(G)

Example 2.2:



D′={v_1,v_3} is the inverse domination set where D′⊆ V-D

The Inverse Domination number of G is γ′(G)=2

Definition: 2.3 [2]

Let G = (V, E) be a graph and u, v∈ V. Then u strongly dominates v if deg u ≥deg v

A set D ⊂ V is called Strong Dominating set of G if every vertex in V-D is strongly dominated by atleast one vertex in D. The strong domination number γ_s(G) is the minimum cardinality of a Strong Dominating set.

Definition: 2.4 [2]

A set D of the vertices in a graph G is called an Independent dominating set of G if D is both an independent and a dominating set of G. This set is also called a Stable set or a Kernel of the graph. The Independent Domination number i(G) is the cardinality of the smallest Independent Dominating set.

Definition: 2.5 [2]

The Independent Strong Domination number i_s(G) of a graph is the minimum cardinality of a Strong set which is also a Independent set.

Definition: 2.6 [3]

Let G=(V,E) be a graph. A set S⊆V(G) is 2-dominating set of G if every vertices of V(G)\S is adjacent to atleast two vertices in S. The 2-Domination number of G is denoted by γ_2(G) is the minimum cardinality of a 2-dominating set of G.

Definition: 2.7 [3]

A set D ⊆V(G) is 2-dominating set of G if every vertices of V(G)\D is adjacent to at least two vertices in D. The 2- dominating set is called Strong 2-Dominating set if degu ≥degv.

The minimum cardinality of Strong 2-Dominating set is denoted by γ_{2s}(G) and is known as Strong 2-Domination number.

Definition: 2.8 [3]

The 2-Dominating set which is independent dominating set is said to be Independent 2-Dominating set. The minimum cardinality of Independent 2-Domination number and is denoted by i_2(G).

Definition: 2.9 [3]

The independent strong dominating set which is 2-dominated is called Independent strong 2-dominating set. The minimum cardinality of independent strong 2-dominating set and is denoted by i_{2s}(G).

PRELIMINARIES

Preposition 1: [4]

Let D be a minimal strong dominating set. Then for each $v \in D$, one of the following holds:

(i) No vertex in D strongly dominates v.

(ii) There exists a vertex $u \in$ V-D such that v is the only vertex in D which strongly dominates u.

3. COCKTAIL PARTY GRAPH

3.1 Introduction

In this section we discussed domination and 2-domination of Cocktail Party Graph.

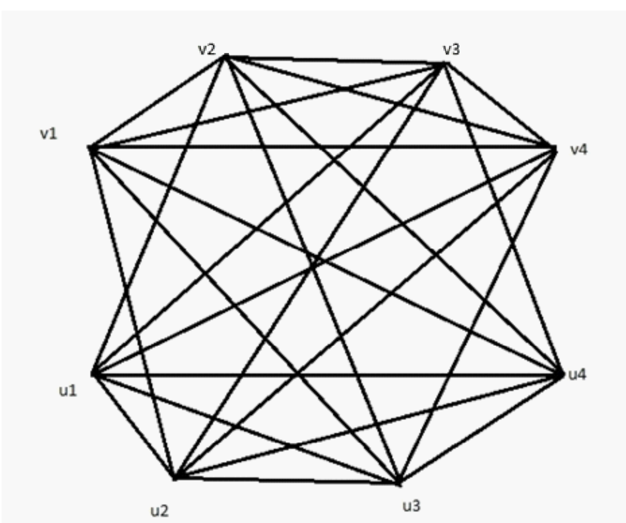
3.2 Cocktail Party Graph: [5]

The Cocktail Party Graph of order n is also called as the hyperoctohedral graph or Roberts graph which consists of 2 rows of paired nodes in which all nodes are paired ones that is connected with an edge.

This graph arises from the Handshake Problem. It is Complete n-partite graph and is denoted by K_{2n}. The Cocktail party graph of order n is isomorphic to the Circulant Graph Ci for all i= 1,2,3,… (2n). The n-Cocktail party graph is also called as the

(2n,n)-Turan graph.

Example: 3.2.1



Adjacency Matrix of the Cocktail Party Graph

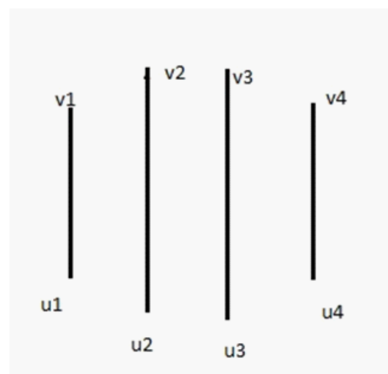
CP_k When k=8

(Example: 3.2.1)

$$A = (aij) = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \left[\begin{array}{cccccccc} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{array}\right] \end{array}$$

3.3 Complement of Cocktail Party Graph When K = 8

(Example: 3.3.1)



Adjacency Matrix of Complement of Cocktail Party CP_k Graph when k=8

(Example: 3.3.1)

$$A' = (b_{ij}) = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \left[\begin{array}{cccccccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

3.4 Domination and 2-Domination Parameters for Cocktail Party Graph

Theorem: 3.4.1

If G is a Cocktail Party graph of order $k = 2n \ \forall \ n \geq 2$ then

i) $\gamma(G) = 2$

ii) $\gamma_2(G) = 2$

iii) $i(G) = 2$

iv) $i_2(G) = 2$

Proof:

Let G be the cocktail party graph with 2n vertices. $V=\{u_1,v_1,u_2,v_2,u_3,v_3,\ldots..u_n,v_n\}$

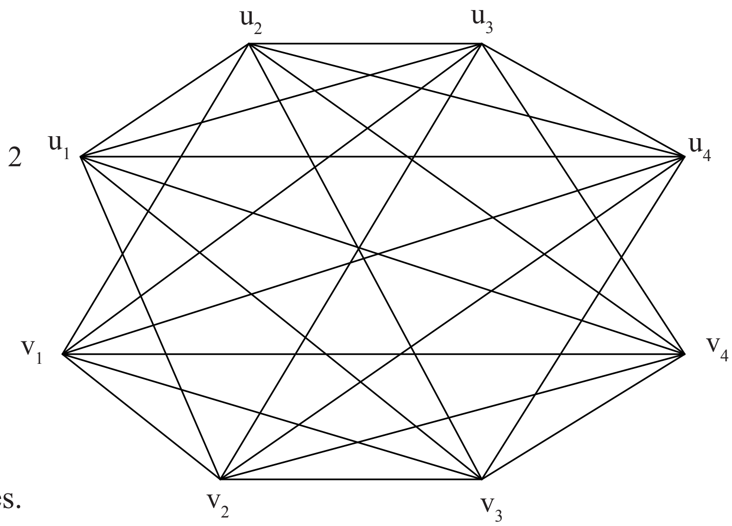
Let $D=\{u_i,v_i\}$ $\forall i=1,2,3,\ldots n$ be the subset of V(G). Since each vertices of D are non-adjacent and also every vertices of (V-D) is adjacent to at least one vertex of D.

By the definition of Dominating set , D is the Dominating set and it is minimum dominating set and the Domination number is $\gamma(G)=2$.

Each vertices of D are non-adjacent. By the definition of Cocktail party graph we know that each vertices of the graph is adjacent to every other vertices except to its paired vertex (i.e) every vertices in D is adjacent to at least 2 vertices of V−D. Then by the definition of 2-dominating set we can say that D is the minimum 2-Dominating set of G. The 2-Domination number is $\gamma_2(G)=2$ Since no two vertices in D are adjacent and every vertices of $V\text{-}D_2$ is adjacent to at least to one vertex of D. By the definition of Independent dominating set, D is the minimum Independent Dominating Set. ∴The Independent Domination Number is $i(G)=2$. Since each vertices of D_3 are non-adjacent ,it is an independent set. It satisfies the definition of independent 2-Dominating set. D is the minimum Independent 2-Dominating set.

∴The Independent 2-Domination number of G $i_2(G)=2$.

Example: 3.1



When n = 2k for k = 4

$V=\{u_1,v_1,u_2,v_2,u_3,v_3,u_4,v_4\}$ is the vertex of graph

$D=\{u_1,v_1\}$ is the subset of V(G) and it is the minimum dominating set of G|D|=2.Then domination number $\gamma(G)=2$.Since each vertices of D dominates two vertices in V-D, D is the minimum 2-dominating set & |D|=2 and 2- Domination number of G is $\gamma_2(G)=2$. the vertices of D are non-adjacent and So D is the independent set. Hence, D is the independent dominating set. The Independent domination number of G is $i(G)=2$.Since it satisfies the condition of Independent 2-Dominating set and so D is the Independent 2-Dominating set. Thus, the Independent 2-Domination number is $i_2(G)=2$.

Theorem: 3.4.2

Suppose G is a Cocktail Party graph of order k=2n then

i) $\gamma_s(G) = 2$

ii) $\gamma_{2s}(G) = 2$

iii) $i_s(G) = 2$

iv) $i_{2s}(G) = 2$

Proof:

Let G be the Cocktail Party graph with 2n vertices and $V=\{u_1,v_1,u_2,v_2,\ldots u_n,v_n\}$

Let $D=\{u_i, v_i\}$ $\forall i=1,2,3,\ldots n$ be the subset of V(G). Since $\deg(u_i)\geq\deg(v_i)$ $\forall u,v\in V$. The vertices in D are strongly dominated by V-D.Then by the definition of Strong Dominating set, D is the minimum Strong Dominating set. So, the Strong Domination number $\gamma_s(G)=2$.

Every vertices of V-D is adjacent to at least 2 vertices of D. D is a 2-Dominating set. By the definition of Strong Domination set, D is the Strong Dominating set of G. Hence, D is the Strong 2-Dominating set .The Strong 2-Domination number is $\gamma_{2s}(G)=2$. Since D is the Strong Dominating set and D is the Independent Strong Dominating set.

$\therefore$ The Independent Strong Domination number is $i_s(G)=2$. Since D is a 2-Dominating set ,the vertices in D are non-adjacent. Then D is the independent 2-Dominating set. Since $\deg(u_i)\geq\deg(v_i)$ $\forall$ u ,v$\in$V. The set D is the Strong Dominating set. Hence D is the independent Strong 2-Dominating set. Thus the Independent Strong 2-Domination number $i_{2s}(G)=2$.

Hence proved.

From example 3.1

$D=\{u_1,v_1\}$ and $V\text{-}D=\{u_2,v_2,u_3,v_3,u_4,v_4\}$

Since $\deg(u_i)\geq\deg(v_i)$. D is the Strong dominating set of G. The Strong Dominating number of G is $\gamma_s(G)=2$. Also D is the independent dominating set and Strong dominating set. We have D is the Independent Strong Dominating set . Independent Strong domination number of G is $i_s(G)=2$. Since $\deg(u_i)\geq\deg(v_i)$. The vertices of V-D is strongly dominated by D and the vertices of V-D is adjacent to at least two vertices of D. Thus D is the strong 2-dominating set. Hence the Strong 2-Domination number is $\gamma_{2s}(G)=2$. D is independent 2-dominating set and also Strong 2-dominating set. Therefore D is an Independent strong 2-Dominating set Independent Strong 2-Domination number is $i_{2s}(G)=2$.

4. MOBIUS LADDER GRAPH

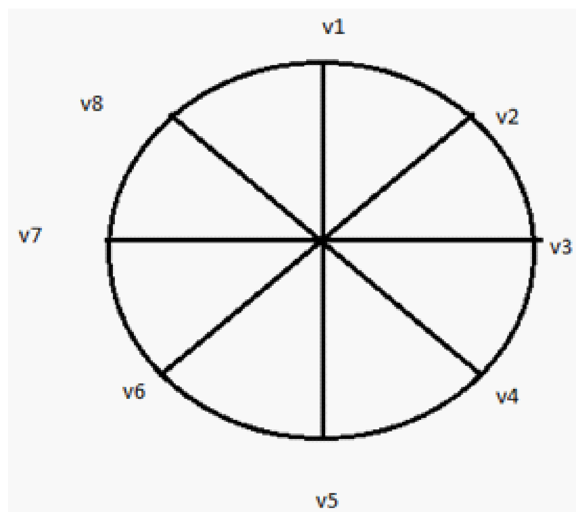
4.1 Introduction

In this section we determined the domination and 2-domination parameters of the MOBIUS LADDER GRAPH.

4.2 Mobius Ladder Graph [5]

The Mobius ladder graph sometimes called as Mobius Wheel of order n .It is a simple graph obtained by introducing a twist in a prism graph of order n that is isomorphic to the Circulant graph $C_{i2n}(1,n)$. It is denoted by Mn. This graph is introduced by Jakobson and Rivinin 1999. Thus Mobius Ladder graph constructed with 1 and n^{th} jump and is said to be Circulant graph. The number of vertices will be 2n.

Example: 4.2.1



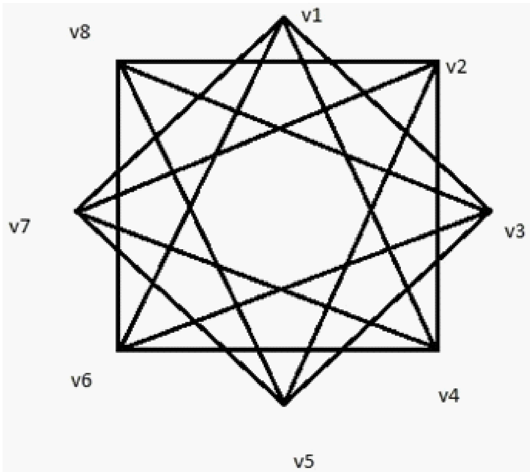
4.3 Adjacency Matrix of Mobius Ladder Graphm$_n$ When N=8

(Example 4.2.1)

$$A=(a_{ij})=\begin{array}{c} \\ v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6\\ v_7\\ v_8 \end{array}\begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array}\\ \left[\begin{array}{cccccccc} 0&1&0&0&1&0&0&1\\ 1&0&1&0&0&1&0&0\\ 0&1&0&1&0&0&1&0\\ 0&0&1&0&1&0&0&1\\ 1&0&0&1&0&1&0&0\\ 0&1&0&0&1&0&1&0\\ 0&0&1&0&0&1&0&1\\ 1&0&0&1&0&0&1&0 \end{array}\right] \end{array}$$

4.4 Complementary Graph of Mobius Ladder Graph

Example: 4.4.1



4.5 Adjacency Matrix of Complement of Mobius Ladder Graph m_n

(Example 4.4.1)

$$A' = (b_{ij}) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \left[\begin{array}{cccccccc} 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{array}\right] \end{array}$$

4.6 Domination and 2-Domination Parameters of Mobius Ladder Graph

Theorem 4.1:[6]

Let G be the Mobius Ladder graph of order n=2k $\forall$ k≥2 then

$\gamma_2(G)=n/2$

Proof:

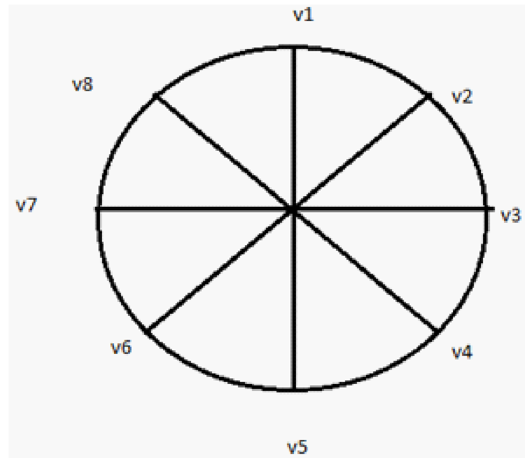
Let G be the Mobius Ladder graph with 2k vertices. $V=\{v_1,v_2,v_3,\ldots\ldots v_k\}$, $\forall$ k≥2

Let $D=\{v_{2i+1}\}$, $\forall i=0,1,2,3,\ldots.n/2$ be the subset of V(G). Since each vertices of D are non-adjacent and each vertices of V-D is adjacent to at least 2 vertices of D. Therefore, D is the minimum dominating and 2-Dominating set and also |D| =n/2.

∴The domination number and 2-Domination number of G is $\gamma_2(G)=n/2$.

Hence proved.

Example:



$V=\{v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8\}$

Let $D=\{v_1,v_3,v_5\}$ is the minimum dominating set of V, |D|=3.

Domination number of graph is γ(G)=3. Suppose D′⊆ (V-D). Then D′ is the dominating set of the graph G. |D′| = 3. The Inverse Domination number of the graph is γ′(G)=3. This is an independent set of graph since the vertices are non- adjacent. We say that D is the Independent Dominating set and |D|=3. The Independent Domination number of the graph i(G)=3. Since deg (v_i)≥ deg (v_{i+1}). Thus D serve as a Strong Dominating set of G. Strong Domination number of a graph is $\gamma_s(G)=3$. D is an Independent dominating set and Strong Dominating set we can say that D is the Independent Strong Dominating set. The Independent Strong Dominating number is $i_s(G)=3$. Let $D_1=\{v_1,v_3,v_5,v_7\}$ is the 2-Domination of G. since $V\text{-}D_1=\{v_2,v_4,v_6,v_8\}$. Since every vertices of $V\text{-}D_1$ is adjacent to at least 2 vertices of D_1.

D_1 is the minimum 2-Dominating set of G and so the 2-Domination number is$\gamma_2(G)=4$.

$D_1'=\{v_2,v_4,v_6,v_8\}$ where $D_1' \subseteq V\text{-}D_1$ is the inverse 2-dominating set of G and $|D_1'|=4$. Thus the Inverse 2-Domination number of graph is $\gamma_2(G)=4$. Though $D_1=\{v_1,v_3,v_5,v_7\}$ is the 2-dominating set but it is not an independent set of graph G and so it is not considered as independent 2-dominating set. Therefore no Independent 2-Dominating set exists.

But $deg(v_i) \geq deg(v_{i+1)}$ ∀i=1,2,…7. D_1 is the Strong 2-Dominating set and $|D_1| = 4$. The Strong 2-Dominating number of the graph is $\gamma_{2s}(G)=4$

Since we don't have any independent 2-dominating set there won't be any independent strong 2-dominating set for the Mobius ladder graph of order 8.

5. CONCLUSION

In this paper we determined domination number, Inverse domination number, Independent domination number, Strong domination number, 2-Domination number, Inverse2-domination number, Independent 2-domination number, Strong 2-domination number, Independent strong 2-domination number for Cocktail party graph and Mobius ladder graph.

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